The Tragedy of Complexity*

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Abstract

This paper presents an equilibrium theory of product complexity. Complex products generate higher potential value, but require more attention from the consumer. Because consumer attention is a limited common resource, an *attention externality* arises: Sellers distort the complexity of their own products to *grab attention* from other products. This externality can lead to *too much* or *too little complexity* depending on product features and the consumer's attention constraint. Products that are well understood in equilibrium are too complex, while products that are not well understood are too simple. Our theory sheds light on the absolute and relative complexity of different goods, including retail financial products.

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1 Introduction

Products, real and financial, differ vastly in their complexity. Some products are exceedingly complicated: Never-ending options in retail financial products, such as insurance policies or retirement plans; endless features and settings for smartphones and software. Other products appear overly simplified, e.g. the media and politicians often seem to oversimplify complex policy issues. But what is the right level of complexity for a good? And does the market deliver the efficient level of complexity? This paper proposes an equilibrium theory of complexity to shed light on these issues.

The key premise of our analysis is that complex products generate higher potential value, but require more of the consumer's limited attention. By allowing complexity to create value, our framework departs from much of the existing literature that has mostly focused on complexity as a means of obfuscation. By explicitly recognizing the consumer's limited attention, our analysis highlights a novel *attention externality*: When choosing the complexity of their goods, sellers do not take into account that attention is a common resource. In equilibrium, sellers therefore distort the complexity of their products, but in doing so they divert attention away from other goods. For example, an insurance company may decide to provide a more complicated, customized health insurance policy. While this can increase the stand-alone value of the health policy, the insurance company does not take into account that the more complicated health insurance policy leaves the consumer with less time to understand products from other sellers, such as her pension plan or her home insurance.

Our analysis yields three main results. First, the presence of the attention externality means that equilibrium complexity is generally inefficient. Specifically, the tragedy of the commons with respect to consumer attention can lead to *too much or too little complexity*, depending on the direction of the consumer's attention reallocation in response to changes in complexity. We refer to this generic inefficiency in complexity choice as the *tragedy of complexity*. Second, we characterize which types of goods end up being too complex and which too simple. Perhaps counterintuitively, products that are relatively well understood tend to be the ones that are too complex relative to the planner's solution, whereas products that are not well understood tend to be too simple. Third, we provide a set of comparative statics for a fully worked-out equilibrium of complexity choices. Among other things, this analysis reveals that equilibrium complexity is more likely to be excessive when attention (per good) is abundant. This leads to a *complexity paradox*: Rather than helping consumers deal with complexity, increases in consumers' information processing capacity can make it more likely that complexity is excessive. The converse of this insight leads to the *curse of variety*: As the number of differentiated goods competing for a given amount of consumer attention increases, this can lead to an inefficient dumbing down of products.

In our model, a consumer with limited attention purchases goods from a number of sellers of differentiated goods. We model limited attention by assuming that the consumer has a fixed time budget that she allocates across all goods. Sellers have market power, so that they extract a share of the value generated by their good, and non-cooperatively choose the complexity of the good they are selling. The consumer's valuation of a good consists of two components. First, it directly depends on the good's complexity. This captures that, all else equal, a more complex good can be worth more to the consumer, for example, because of additional features, functionality or customization. Second, the consumer's valuation is higher the more time she spends on understanding the good. Therefore, as in Becker (1965), the consumer's time acts as an input to the value of consumption goods. In particular, a deeper understanding of the good allows the consumer to make better use of the good's features, functionality, or customization. More complex goods require more attention to achieve the same depth of understanding. Specifically, we assume that when the complexity of a good doubles, it takes the consumer twice as much time to reach the same depth of understanding. The buyer's understanding of a good therefore depends on the *effective attention* (time spent divided by complexity) paid to the good. The assumption that more complex goods are harder to understand leads to a trade-off: A more complex good is potentially more valuable to the consumer, but the consumer also has to pay more attention to reach the same depth of understanding.

When sellers choose the complexity of their good, they internalize that consumers respond by increasing or decreasing the attention allocated to their good. Sellers therefore have an incentive to distort the complexity of their good to increase the amount of attention paid to it by the buyer. The reason is that more attention from the consumer increases the value of the good, some of which is extracted by the seller. However, sellers do not internalize that attention is a common resource—an increase in attention paid to their good necessarily corresponds to a decrease in attention paid to other goods. These other goods decrease in value, resulting in an *attention externality*.

While, in principle, the direction of the externality depends on the characteristics of all the goods in the economy, we show that there is a simple test to determine whether a seller has an incentive to increase or decrease the complexity of his own product relative to the social optimum. In particular, a seller has an incentive to increase the complexity of his good beyond the level that a planner would choose if and only if a consumer who keeps the attention she pays to the good fixed is worse off after the change. This effect, which we call the *software update effect* (i.e., the feeling that additional features have made a product worse) is therefore a red flag for an inefficient increase in complexity.

Equilibrium complexity can feature too much or too little complexity, depending on whether sellers attract attention away from other goods by raising or lowering the complexity of their own good. Our model shows that the consumer's depth of understanding of a good is a simple a diagnostic for whether a good is likely to be overly complicated. Goods that are well understood in equilibrium (i.e., high effective attention) are too complex, whereas goods that are not well understood (low effective attention) are too simple. The reason is that it is goods of intermediate complexity that attract the most attention from consumers: Simple goods are well understood even when little attention is paid to them, whereas very complex goods cannot be understood even if the consumer devotes all of her attention to them. To attract the consumer's attention, the seller therefore distorts complexity towards intermediate levels—increasing the complexity of goods that should be relatively simple and decreasing the complexity of goods that should be relatively complex.

Our model generates a number interesting comparative statics. For example, paradoxically, goods tend to be overly complex precisely when the buyer has a relatively large attention budget. Therefore, rather than helping buyers deal with the complexities of everyday life, improvements in information processing capacity may therefore be a driver of excessive complexity—the *complexity paradox*. In contrast, when more goods compete for a fixed amount of buyer attention, goods can end up being inefficiently simple, an effect we call the *curse of variety*. Our model therefore provides a potential explanation for why the recent increase of online and social media outlets has gone hand in hand with a dumbing down of content.

Related literature. By viewing time as an input to the value of consumption goods, our approach to modeling complexity builds on the classic work of Becker (1965). We extend this framework by introducing complexity choice. The choice of complexity affects the value of the good directly, but also how the consumer transforms her time into understanding the good. By assuming a limited time budget for the consumer, our framework captures that complexity is inherently a bounded rationality phenomenon (Brunnermeier and Oehmke, 2009). The constraint on the consumer's time serves a role similar to information processing constraints in information theory (e.g., Sims, 1998, 2003).

Our approach to complexity differs from most of the existing literature, which has focused on complexity as a means to obfuscate in order increase market power or influence a consumer's purchasing decision (Carlin 2009, Carlin and Manso 2010, Piccione and Spiegler 2012, Spiegler 2016, and Asriyan et al. 2018). In contrast to this literature, in our model complexity is value enhancing, at least to some extent. Moreover, in our framework the cost of complexity is not an increase in market power or a distortion in the consumer's purchasing decision. Rather, it manifests itself an externality that the complexity of one good imposes on the equilibrium value of other goods.

A key aspect of our paper, competition for attention, is studied also by Bordalo et al. (2016). In contrast to our paper, their focus is on the salience of certain product attributes: Consumer attention can be drawn to either price or quality, resulting in equilibria that are price- or quality-salient. Despite the difference in focus, their analysis shares with ours an attention externality across goods. Finally, our work is related to the literature on providing default options, see Choi et al. (2003). Specifically, privately optimal excess complexity may explain why sellers are often unwilling to provide default options that would make the product less time consuming to use.

2 Model Setup

2.1 Model setup

We consider an economy with N sellers (he) and a single buyer (she). Goods are differentiated and there is one seller per good $i \in \{1, ..., N\}$, each endowed with an indivisible unit of good i. Because goods are differentiated, sellers have some market power, which we capture in reduced form by assuming that seller i can extract a share $\theta_i \in (0, 1)$ of the value v_i generated by good i, while the buyer gets to keep the remaining $1 - \theta_i$ share.[1]

The key decision for each seller is to choose the complexity c_i of the good he sells. While complexity has no direct cost (or benefit) for the seller, complexity matters because it affects the value of the good. On the one hand, complexity can add value, for example, when it arises as a byproduct of customization that caters the buyer's specific needs. On the other hand, realizing the full value of a more complex good requires attention from the buyer, who needs to devote time to understand the precise nature of the more complex, customized good.[2] The total value of a complex good therefore arises from the combination of the characteristics of the good itself and the time that the buyer allocates to the good. In this respect, our paper builds on classic work on time as an input into the utility derived from market goods pioneered by Becker (1965).

[1] For now we simply assume this sharing rule. We provide a more detailed discussion of this and other assumptions in Section 3.5.1.

[2] Attention may be devoted to the good before purchase (e.g., figuring out the specific features of a more complex product) or during the use of the good (e.g., when the use of a more complex good is more time consuming). Our model can accommodate both interpretations.

To capture these features of complexity more formally, we assume that the value to the buyer of consuming a unit of good i with complexity c_i, having allocated t_i units of time to good i, is given by

$$v_i\left(c_i, \frac{t_i}{c_i}\right), \tag{1}$$

which we assume is twice continuously differentiable in both arguments. The first argument of $v_i(\cdot,\cdot)$ captures that the value of the good depends directly on the complexity of the good. We assume that for sufficiently low levels of complexity $\frac{\partial v_i}{\partial c_i} > 0$, such that *ceteris paribus* some complexity raises the value of the good. However, as a good becomes more complex, the direct benefit of complexity exhibits diminishing marginal returns, $\frac{\partial^2 v_i}{\partial c_i^2} < 0$. At some point, the marginal value of complexity could even turn negative.

The second argument of $v_i(\cdot,\cdot)$ reflects that the value of the good to the buyer increases with the attention (measured in units of time) that the buyer allocates to the good. However, as complexity increases, a unit of attention becomes less valuable. What matters, rather, is effective attention, which we define as time spent on the good divided by the good's complexity, t_i/c_i (we sometimes simply denote effective attention by $e_i \equiv t_i/c_i$). Effective attention also measures how much the buyer understands the good, for this reason, we also refer to it as the depth of understanding. This assumption captures that more complex goods take more time to understand and make the best use of. In our particular specification, where effective attention equals time spent divided by complexity, a good that is twice as complex takes twice as much time to understand to the same extent, e.g. a contract twice as long takes twice as much time to read. We make standard assumptions on the effect of effective understanding on the value of the good: All else equal, a deeper understanding increases the value of the good to the buyer, $\frac{\partial v_i}{\partial (t_i/c_i)} > 0$, but with diminishing marginal returns, $\frac{\partial^2 v_i}{\partial (t_i/c_i)^2} < 0$.

In addition to the relatively standard assumptions on positive but diminishing returns to effective attention a good, we make two additional assumptions on how the consumer's understanding affects the value of the good.

Assumption 1. *The marginal value of understanding the good, t_i/c_i, is independent of the level of complexity:* $\frac{\partial^2 v_i}{\partial c_i \partial(t_i/c_i)} = 0$.

Assumption 2. *The value of good i is bounded above and below in the consumer's effective attention:* $v_i(c_i, 0) > 0$ *and* $v(c_i, \infty) < \infty$.

Assumption 1 states that the marginal value of the consumer's effective attention is independent of the complexity of the good. Therefore, when a good is twice as complex, the buyer has to spend twice as much time on the good to gain the same level of understanding. Assumption 2 implies that good i is valuable even when buyers pay no attention to it, $v_i(c_i, 0) > 0$, and that the value of the good remains bounded even when effective attention (the depth of understanding) is infinite, $v_i(c_i, \infty) < \infty$. The first part of this assumption guarantees that all goods are consumed in equilibrium. The second part ensures that very simple goods ($c_i \to 0$) have finite value, even when these goods are extremely well understood ($t_i/c_i \to \infty$).

The key decision faced by the buyer is which goods to consume and how much attention $t_i \geq 0$ to allocate to each of these goods. In making her decision, the buyer takes the complexity of each good as given, but takes into account that she receives a share $1 - \theta_i$ of the value v_i generated by good i. The key constraint faced by the buyer is that her attention is limited. Specifically, the buyer has a fixed amount of time T that she can allocate across the N goods. One interpretation of this limited attention constraint is that it introduces an element of bounded rationality that is required to make complexity meaningful (Brunnermeier and Oehmke, 2009). We assume that the buyer's utility is quasi-linear in the benefits derived from the N goods and wealth, and that the buyer is deep pocketed. This assumption implies that our results are driven by the buyer's attention constraint (introduced in more detail below) rather than a standard budget constraint.

3 The Tragedy of Complexity

In this section, we present the main conceptual result of our paper: equilibrium complexity is generally inefficient. We first characterize the buyer's attention allocation problem and then contrast the sellers' privately optimal complexity choice to that chosen by a benevolent planner.

3.1 The Buyer's Problem

By Assumption 2, the buyer receives positive utility from consuming good i even when paying no attention to it. It is therefore optimal for the buyer to purchase all N goods. The buyer's maximization problem therefore reduces to choosing the amount of attention she allocates to each good (taking as given complexity c_i)

$$\max_{t_1,..t_N} \sum_{i=1}^{N} (1-\theta_i) \cdot v_i \left(c_i, \frac{t_i}{c_i} \right), \tag{2}$$

subject to the attention constraint[3]

$$\sum_{i=1}^{N} t_i \leq T. \tag{3}$$

Using λ to denote the Lagrange multiplier on the attention constraint, the buyer's first-order condition is given by

$$(1-\theta_i) \cdot \frac{\partial v_i \left(c_i, \frac{t_i(c_1,\ldots,c_N)}{c_i} \right)}{\partial \left(\frac{t_i}{c_i} \right)} \cdot \frac{1}{c_i} \leq \lambda, \tag{4}$$

which holds with equality when $t_i > 0$. The first-order condition states that, if the buyer pays attention to the good, the marginal value of an additional unit of attention to good i must equal the shadow price of attention λ. Because the buyer can only extract a fraction $1-\theta_i$ of the value generated by good i, all else equal, it is optimal to allocate more time to goods for which this fraction is large.

[3] By rewriting the attention constraint as $\sum_{i=1}^{N} \frac{t_i}{c_i} \cdot c_i \leq T$ (i.e., multiplying and dividing by c_i), we see that one can think of the attention constraint as a standard budget constraint, where the good purchased by the buyer is effective attention t_i/c_i, the price of effective attention for good i is the complexity of that good, c_i, and the buyer's wealth is her endowment of time, T. We show in Section 3.5.3 that this interpretation can be useful because it allows us to draw parallels to classic results from consumer demand theory.

3.2 Equilibrium Complexity: The Seller's Problem

Seller i's objective is to maximize profits, given by a fraction θ_i of the value generated by good i. The seller's only choice variable is the complexity c_i of his good. However, in choosing c_i, the seller internalizes that his complexity choice affects the amount of attention allocated to the good by the buyer. Like a Stackelberg leader, the seller internalizes that the attention the buyer pays to his good, $t_i(c_1, \ldots, c_N)$, is function of c_i. The seller's objective function is therefore

$$\max_{c_i} \theta_i \cdot v_i \left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i} \right), \tag{5}$$

with an associated first-order condition of

$$\theta_i \cdot \frac{d}{dc_i} v_i \left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i} \right) \leq 0. \tag{6}$$

Again, this first order condition holds with equality whenever $c_i > 0$. Assuming that c_i is indeed an internal solution[4] and taking the total derivative, the first-order condition (6) can be rewritten as

$$\frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial c_i} = \frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial \left(\frac{t_i}{c_i} \right)} \cdot \frac{t_i}{{c_i}^2} - \frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial \left(\frac{t_i}{c_i} \right)} \cdot \frac{1}{c_i} \cdot \frac{\partial t_i}{\partial c_i}. \tag{7}$$

This optimality condition states that, from the seller's perspective, the optimal level of complexity equates the marginal increase in value from additional complexity (the left-hand side of Equation (7)) to the value reduction that arises from lower levels of effective attention holding the buyer's attention to the good constant (the first term on the right-hand side), net of the change in the good's value that arises from the buyer's change in attention paid to good i in response to an increase of the complexity of that good (the second term on the right-hand side). In equilibrium, this first-order condition must hold for each seller i.

[4] A sufficient condition for $c_i > 0$ is that a standard Inada condition holds with respect to complexity.

The key observation is that sellers take into account that a change in the complexity of their good changes the amount of attention that the buyer will allocate to the good, as indicated by the term containing $\frac{\partial t_i}{\partial c_i}$ in Equation (7). In particular, sellers perceive the additional attention paid to their good in response to a change in complexity as a net gain, even though in aggregate changes in attention paid are merely a reallocation—any additional attention paid to good i would otherwise be allocated to goods of other sellers. Because the seller of good i is essentially diverting attention away from other goods, we refer to this as the *attention grabbing* effect.

Using the buyer's first-order condition (4), we can rewrite the seller's optimality condition (7) in terms of the shadow price of attention λ, which for $c_i > 0$ and $t_i > 0$ yields

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta_i}\left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right). \tag{8}$$

Rewriting the first-order condition in this more concise way is useful when comparing the seller's first-order condition to the planner's optimality condition derived in the next section.

3.3 Optimal Complexity: The Planner's Problem

We now turn to the planner's choice of product complexity. The key difference compared to the seller's profit-maximization problem described above is that the planner takes into account that the buyer optimally allocates attention across all goods. Therefore, the planner internalizes the effect of a change in the complexity of good i not only on the value of good i but also, via the buyer's attention reallocation, on all other goods $j \neq i$.

More formally, the planner chooses the product complexities of all N goods to maximize total value,

$$\max_{c_1,\ldots c_N} \sum_{i=1}^{N} v_i\left(c_i, \frac{t_i(c_1,\ldots,c_N)}{c_i}\right). \tag{9}$$

Following the same steps as in the derivation for the seller's first-order condition (including the assumption that c_i^* is an internal solution), the optimality condition for the planner's complexity choice

c_i^* for good i is given by

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{t_i}{c_i^2} - \sum_{j=1}^{N} \frac{\partial v_j\left(c_j, \frac{t_j}{c_j}\right)}{\partial\left(\frac{t_j}{c_j}\right)} \cdot \frac{1}{c_j} \cdot \frac{\partial t_j}{\partial c_i}. \tag{10}$$

This optimality condition highlights the difference between the sellers' complexity choice (8) and the planner's solution. In particular, whereas the seller of good i only takes into account the change in the valuation of good i that results from the reallocation of attention to or from good i, the planner takes into account the changes in valuation that results from the reallocation of attention across all goods, resulting in $N-1$ additional terms on the right hand side. Therefore, the privately optimal complexity choice generally differs from the planner's solution—reallocation of attention from other goods to good i results in an externality that is not taken into account by the seller of good i.

As before, using the buyer's first-order condition (4), we can rewrite the planner's optimality condition (10) in terms of the shadow price of attention λ, which yields

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta_i} \cdot \left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right) - \sum_{j \neq i} \frac{\lambda}{1-\theta_j} \cdot \frac{\partial t_j}{\partial c_i}, \tag{11}$$

where the second term on the right hand side captures the externality that is neglected by the seller.

A particularly simple case arises when all sellers have equal market power, such that $\theta_i = \theta$. In this case, the planner's optimality condition reduces to

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta} \cdot \left(\frac{t_i}{c_i} - \sum_{j=1}^{N} \frac{\partial t_j}{\partial c_j}\right) = \frac{\lambda}{1-\theta} \cdot \frac{t_i}{c_i}, \tag{12}$$

where the last step makes use of the fact that, when viewed across all goods, attention is merely reallocated (i.e., $\sum_{j=1}^{N} t_j = T$ implies that $\sum_{j=1}^{N} \frac{\partial t_j}{\partial c_j} = 0$).

3.4 The Complexity Externality

A comparison between the seller's and the planner's first-order condition reveals that there is an externality in complexity choice. The seller of good i has an incentive to deviate from the socially optimal complexity c_i^* whenever there is a private benefit from doing so. Under equal market power of sellers ($\theta_i = \theta$), a simple comparison of the first-order conditions (8) and (12) shows that this is the case whenever at the optimal complexity c_i^* the attention grabbing effect is nonzero, $\frac{\partial t_i}{\partial c_i} \neq 0$. When $\frac{\partial t_i}{\partial c_i} > 0$ the seller of good i has an incentive to increase the complexity of his good beyond the socially optimal level, whereas when $\frac{\partial t_i}{\partial c_i} < 0$ the seller if good i wants to decrease complexity below the socially optimal level. In both cases, the direction of the externality is therefore driven by the desire to divert the buyer's attention away from other goods. In fact, while this result is seen most easily under equal market power for sellers, the result is true also when market power differs across sellers, as shown formally in the following proposition.

Proposition 1. The Tragedy of Complexity. *Starting from the planner's solution $(c_1^*, ...c_N^*)$ and keeping the complexity of all other $j \neq i$ goods fixed at c_j^*, the seller of good i*

(i) has an incentive to increase complexity c_i above its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i} > 0$;

(ii) has an incentive to decrease complexity c_i below its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i} < 0$;

(iii) has no incentive to change complexity c_i from its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i}$=0.

Proposition 1 states that the complexity externality has the same sign as the attention grabbing effect: Sellers have a (local) incentive to increase the complexity of their good beyond its optimal level if buyers respond by increasing the amount of attention paid to the good. In contrast, if buyers respond by decreasing the amount of attention allocated to the good when its complexity increases, sellers have a (local) incentive to decrease the complexity of their product below the socially optimal level. Note, however, that the equilibrium distortion is not necessarily in the same direction as local incentive to distort $\frac{\partial t_j}{\partial c_i}$ at the socially optimal complexity level because of wealth effects that work through the shadow price of attention λ. We provide a full analysis of equilibrium complexity choices for a specific functional form for $v(\cdot,\cdot)$ in Section 5.

Whereas Proposition 1 describe the externality in complexity choice in terms of the attention grabbing effect $\frac{\partial t_i}{\partial c_i}$, there are a number other indicators than can equally be used as sufficient statistics to characterize the direction of the externality. In particular, as stated in Lemma 1 below, one can equivalently look at (1) the effect of a change in complexity on the shadow cost of attention, (2) the attention grabbing effect given a fixed shadow cost of attention, and (3) a simple complementarity condition between complexity and buyer attention. To state these results concisely, it is useful to introduce some additional notation. First, using Equation (4) it will sometimes be useful to write attention as a function of the good's own complexity and the shadow cost of attention. We will denote this function by $\tilde{t}_i(c_i, \lambda)$. Second, for the last equivalence result we will rewrite the value of good i in terms of attention instead of effective attention (i.e., we define $\tilde{v}(c,t) = v(t, t/c)$).

Lemma 1. Attention Grabbing: Equivalence Results. *For any given vector of product complexities* $(c_1, \ldots, c_N)$*, the following have the same sign:*

(i) the attention grabbing effect for good i, $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$;

(ii) the effect of good i*'s complexity on the shadow cost of attention,* $\frac{\partial \lambda(c_1,..c_N)}{\partial c_i}$;

(iii) the attention grabbing effect for good i*, keeping the shadow cost of complexity fixed,* $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}\Big|_{\lambda}$;

(iv) the complementarity (substitutability) of attention and complexity, $\frac{\partial^2 \tilde{v}(c_i,t_i)}{\partial c_i \partial t_i}$.

Statement (ii) in Lemma 1 provides intuition for why attention grabbing is an externality: The observation that the externality works through the shadow price also highlights the importance of the assumption of limited attention. If attention could be bought or sold at a fixed cost (i.e., if λ is independent of the sellers' complexity choices), there would be no externality, since increasing the amount of attention allocated to one good would not mean that the attention paid to other goods has to diminish. Statement (iv) in Lemma 1 provides a useful microeconomic interpretation of the complexity externality: There is an incentive for sellers to increase complexity beyond the optimal level when attention and complexity are complements. In contrast, when attention and complexity are substitutes, sellers have an incentive to decrease complexity below the optimal level.

While the complexity externality can lead to too much or too little externality, there is a simple diagnostic that allows us to determine whether an increase in complexity is inefficient. To do so, it is useful to rewrite the seller's first-order condition, dividing it into two parts: The effect of an increase in complexity *holding fixed the buyer's attention* and the additional effect that results from attention reallocation. Written in these terms, the first-order condition (7) (assuming an internal solution) becomes

$$\frac{\partial \tilde{v}_i(c_i, t_i)}{\partial c_i} + \frac{\partial \tilde{v}_i(c_i, t_i)}{\partial t_i} = 0. \tag{13}$$

Rewriting the second part in terms of $v(c_i, \frac{t_i}{c_i})$, we obtain

$$\frac{\partial \tilde{v}_i(c_i, t_i)}{\partial c_i} + \frac{1}{c_i} \cdot \frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{\partial t_i}{\partial c_i} = 0, \tag{14}$$

which shows that the effect of increased complexity holding fixed the buyer's attention has the opposite sign to the attention reallocation effect $\frac{\partial t_i}{\partial c_i}$ (recall that, by assumption, $\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)}$ is strictly positive). This leads to the following Proposition.

Proposition 2. The Software Update Effect. *The seller can attract more attention to its good in equilibrium by increasing complexity ($\frac{\partial t_i}{\partial c_i} > 0$) if and only if the value of good i to the buyer decreases when time allocated to the good is constant, i.e., $\frac{\partial \tilde{v}_i(c_i, t_i)}{\partial c_i} < 0$.*

Proposition 2 explains a familiar phenomenon: Often an updated product initially appears worse than before. For example, following the release of a new version of Excel a user complained: *"I hate the new product I bought. It has far too many features that I will never use and cannot get rid of. [...] Why do u have to make things so difficult?"* Another user replied: *"That's normal. Many people* *found that the new interface in Excel 2007 was a nightmare [... However,] there are so much cool functions and features added. Just take some time to adapt to the new interface."*[5]

Our model reconciles these seemingly contrasting views: Without investing more time it often seems as if the product has become worse than it was before. As stated in Proposition 2, these are exactly the instances when the seller has and incentive to choose excessive complexity. Moreover, our model rationalizes why, despite the apparent reduction in value that arises when attention is held constant, the seller engages in this type of behavior. Once we account for the extra attention allocated to the good by the consumer in response to the change in complexity, the valuation of the good increases. Some of this additional value is then extracted by the seller. The flip side, not internalized by the seller, is that the extra time allocated to the good is taken away from other goods, so that the valuation of those goods decreases. Overall, after the reallocation of attention, the good in question is worth more to both the buyer and the seller, but the value of all other goods suffers to an extent that the buyer is worse off overall. In line with the example above, we refer to the result in Proposition 2 as the software update effect.

3.5 Discussion

In this section we provide a little more discussion of the key assumptions and results of our analysis. Section 3.5.1 provides some further discussion of some of our key modeling assumptions. Section 3.5.2 compares the tragedy of complexity to a traditional tragedy of commons. Section 3.5.3 uses tools from classic consumer demand theory to sign the complexity externality.

3.5.1 Discussion of Key Modeling Assumptions

To keep our model simple yet parsimonious, we have made a number of assumptions. One key simplifying assumption is that the seller keeps an exogenous fixed share of the value of the good. This assumption captures, in reduced form, that sellers of differentiated goods have market power. However,

[5] https://answers.microsoft.com/en-us/msoffice/forum/all/why-is-excel-so-complicated/a2fc9495-1fb6-4bf0-965a-07c2b037606b (August 14, 2015), last accessed November 17, 2018.

our results do not rely on this particular setting. Rather, the crucial underlying assumption is that the seller can extract some of the increase in the value of the good that results when the buyer pays more attention to it. In the following we discuss three different settings which this assumption captures well. First, in non-market transactions, our assumption is equivalent to assuming a benevolent seller (or provider) who is interested in the surplus generated by the good. Examples of such settings include financial regulators that design regulations to maximize the value generated by the markets they regulate. Our model then implies that, in a setting with competing regulators (e.g., financial regulation in the U.S.), the complexity of financial regulation generally does not coincide with the social optimum. Second, if sellers and buyers bargain over the price of a good, our model is equivalent to a setting in which buyers have to pay attention to understand the good before bargaining over the price. In this case, θ_i corresponds to seller i's Nash bargaining weight vis-à-vis the buyer. This interpretation captures many contracting settings in which the buyer has to allocate attention and make contractual choices before signing the contract (e.g., choosing a custom-tailored insurance contract or pension product). Third, the surplus-sharing rule we assume can be micro-founded from a setting in which the good's price is determined by market clearing. Specifically, if the buyers decide how much of each good to consume, market clearing leads to the same equilibrium conditions of complexity choice as in our model.

Another important assumption is that limited attention takes the form of a hard constraint on the consumer's time budget. This is meant to capture the limited amount of time (after work and rest) that a retail consumer can spend on thinking about and consuming goods. This assumption seems fitting for settings that involve retail consumers, such as retail financial products. In the case of companies, one may argue that by employing more people the company can relax its attention constraint. However, as long as expanding attention is associated with an increasing cost (a reasonable assumption given the technological and organizational costs involved), the implications of such a model are equivalent to those with a hard attention constraint. As shown in Lemma 1, the key force that leads to the complexity externality is that complexity choice affects the shadow price of attention.

Finally, we make a key assumption about timing: complexity is chosen before the buyer makes her choices. This results in market power for the buyer, similar to that of a Stackelberg leader. Here, the crucial assumption is not the specific timing, but that the buyer cannot choose (or shop around for) the complexity she prefers. In many markets this timing seems realistic, given that goods and services are often designed before they reach the market and their complexity cannot be easily altered afterwards.

3.5.2 The Tragedy of Complexity and the Tragedy of Commons

The difference between the equilibrium complexity choice and the planner's solution has parallels with the classic tragedy of the commons. Like grass on a common grazing meadow, attention is a shared resource that is used by all goods. However, as we saw above, attention grabbing can manifest itself in *too much or too little complexity*, depending on whether "overcomplicating" or "dumbing down" leads to an increase in buyer attention paid to a particular product. Whereas the complexity externality can go either way, the scarce resource of attention is always overused irrespective of the direction of the externality. Competition for the buyer's attention implies that the shadow price of attention is higher in equilibrium than it would be under the planner's solution, $\lambda^e \geq \lambda^*$, with strict inequality whenever $c_i^e \neq c_i^*$ for at least one good. In words, the buyer constantly feels short of time when sellers compete for her attention.

Proposition 3. The Buyer is Short of Time. *Suppose that equilibrium and optimal complexity differ for at least one good. Then the equilibrium shadow price of attention strictly exceeds the shadow price of attention under the planner's solution,* $\lambda^e > \lambda^*$.

Thus the conventional tragedy-of-commons intuition holds for the fixed-supply common resource used by all goods, attention. The contribution of our paper is to show that the classic tragedy of commons with respect to consumer attention leads to equilibrium complexity that is generically inefficient and can be above or below the efficient the efficient complexity level—the *tragedy of complexity*.[6]

3.5.3 Complexity through the lens of demand theory

The conditions that lead to excess complexity can be cast in the language of consumer demand theory. For simplicity, we demonstrate this in a two-good setting. Rewriting the attention constraint in terms of effective attention, $e_i = \frac{t_i}{c_i}$, we obtain

$$c_1 e_1 + c_2 e_2 = T. \tag{15}$$

This version of the attention constraint shows that we can think of product complexity as the price of a unit of effective attention. Under this interpretation, we can then express the buyer's choice of effective attention for good $i = 1, 2$ as

$$e_i(c_1, c_2, T), \tag{16}$$

the Marshallian demand for effective attention as a function of the complexity of the two goods, c_1 and c_2, and the attention budget, T.

We can now use standard concepts from consumer demand theory to characterize when excess complexity emerges in equilibrium. Suppose seller 1 increases the complexity of his good. Now consider a Slutsky decomposition that divides the change in effective attention the buyer allocates to good 2, $\frac{\partial e_2(c_1,c_2,T)}{\partial c_1}$, into a substitution effect and an income effect. The substitution effect results in a reallocation of effective attention from good 1 to good 2. When the price of effective attention for good 1 is increased, the buyer optimally increases the effective attention paid to good 2. The income effect, on the other hand, results in a decrease in effective attention paid to both goods. When the income effect outweighs the substitution effect, then the increase in the complexity of good 1 leads

[6]What type of policy intervention would solve the tragedy of complexity? According to the above analysis, a regulation that simply aims to reduce complexity is not the right policy. After all, complexity can be too high or too low. Rather, the optimal regulation would have to induce sellers to internalize the shadow cost of attention. In principle, this could be achieved via tradable permits for attention, although such a policy seems difficult to implement.

to reduction in the effective attention paid to good 2. Because c_2 is unchanged, this implies that t_2 decreases and t_1 increases (because $t_1 + t_2 = T$). Therefore, excess complexity arises ($\frac{\partial t_1}{\partial c_1} > 0$) if and only if the income effect for effective attention outweighs the substitution effect.[7]

4 An Explicit Characterization

In this section, we use an explicit functional form for the value of the good to characterize the direction of the complexity externality and the resulting equilibrium. Specifically, we assume that the value of good i is given by:

$$v_i\left(c_i, \frac{t_i}{c_i}\right) = w_i \cdot \left(f_i(c_i) + \delta_i \cdot \frac{\frac{t_i}{c_i}}{1 + \frac{t_i}{c_i}}\right). \tag{17}$$

Under this functional form, the direct benefit from complexity (assuming zero time spent on the good) is given by $f_i(c_i)$. In addition to this direct benefit, the value of good is increasing in the consumer's effective attention, t_i/c_i. The remaining parameters will be useful for comparative statics: w_i captures the utility weight of good i in the buyer's consumption basket, while δ_i captures the importance of understanding the good. Note that this functional form satisfies all assumptions we made above, including Assumptions 1 and 2.

In conjunction with a quadratic benefit function,

$$f_i(c_i) = \alpha_i \cdot c_i - c_i^2, \tag{18}$$

the above functional form allows us to solve for equilibrium attention allocation and complexity in closed form. The parameter α_i captures the direct benefit of complexity for good i. Note that, given the quadratic nature of the benefit function, increasing complexity beyond some level reduces the

[7]Alternatively, one can show that complexity is excessive when the demand for effective inattention is inelastic. Under the interpretation of complexity as the price of a unit of effective attention, the time spent on good i is equal to the buyer's expenditure on that good (i.e., $t_i = c_i e_i$). A standard result from consumer demand theory is that an increase in the price of good i leads to an increase in the total expenditure on that good if and only if the own-price demand elasticity of that good is smaller than one, $\eta_i = \left|\frac{\partial e_i}{\partial c_i}\frac{c_i}{e_i}\right| < 1$.

value of the good to the buyer. Therefore, even without a constraint on buyer attention, the optimal level of complexity of good i is finite (without a constraint on attention, the value of the good would be maximized by choosing $c_i = \frac{\alpha_i}{2}$).

The key to signing the complexity externality is to understand how attention allocated to good i changes when this seller changes the complexity of his good c_i, keeping the complexity of all other goods unchanged. We are therefore interested in the shape of the function $t_i(c_i)$. Recall from Lemma 1 that holding λ fixed does not change the sign of the slope of the function $t_i(c_i)$, so that it is sufficient to characterize the shape of the $\tilde{t}_i(c_i, \lambda)$ in c_i. Focusing on $\tilde{t}_i(c_i, \lambda)$ is useful, because it allows us to derive an explicit expression for the amount of attention paid to good i. Substituting the functional form (17) into the consumer's first-order condition (4), we find that

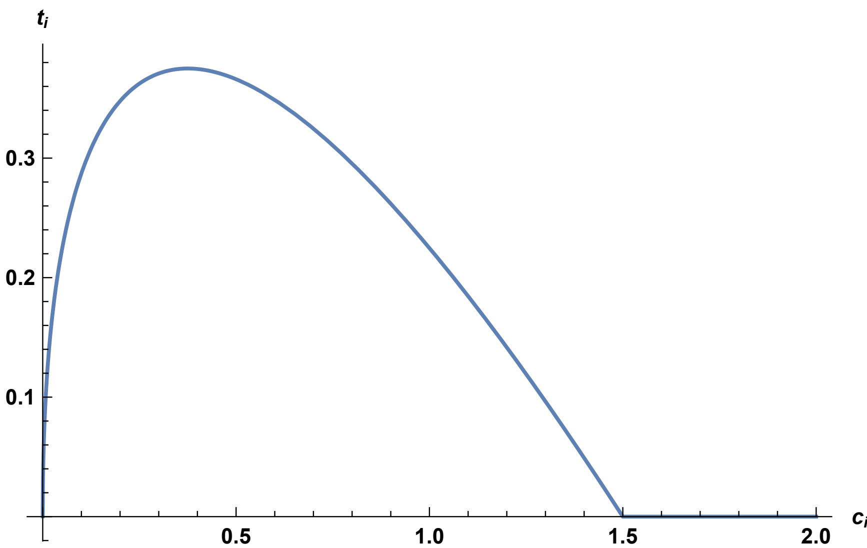
$$\tilde{t}_i(c_i, \lambda) = \max\left(0, \sqrt{\frac{c_i \cdot \delta_i \cdot (1-\theta_i) \cdot w_i}{\lambda}} - c_i\right). \tag{19}$$

Figure 1 plots $\tilde{t}_i(c_i, \lambda)$ as a function of the complexity of good i, holding the shadow cost of attention λ fixed. As we can see, attention follows a hump shape. For low levels of complexity, an increase in the complexity of good i leads to an increase in the attention paid to the good ($\frac{\partial t_i}{\partial c_i} > 0$). In this region, the seller of good i has an incentive to increase the complexity of his good. For higher levels of complexity, the direction of the externality reverses, and an increase in the complexity of good i leads to a reduction in attention paid to good i ($\frac{\partial t_i}{\partial c_i} < 0$), so that the seller of good i has an incentive to decrease the complexity of his good. Finally, above some critical level of complexity, the buyer gives up on learning and pays no attention to good i (even though she still buys the good).[8] Even if the buyer were to allocate all of her attention to the good, she would not understand it well, so that it becomes preferable for the consumer to focus her attention on other goods. In this region,

[8]Real world examples of this phenomenon include the terms and conditions often associated with online purchases or software updates, both classic instances of information overload (see Brunnermeier and Oehmke (2009)).

that we call the *giving up* region, there is no externality, so that equilibrium complexity coincides with the social optimum.[9]

Figure 1: **Attention as a function of complexity (with fixed shadow cost of attention)**



This figure illustrates the buyer's attention choice t_i as a function of the complexity of good i, c_i, holding fixed the shadow price of attention λ. Attention allocation is hump shaped: Initially, $t_i(c_i)$ is increasing, then decreasing, and at some point the buyer chooses to pay no attention to good i. The figure assumes homogenous goods, with parameters $N = 2$, $\delta = 0.9$, $w = 1$, $\alpha = 2$, $\theta = 0.5$, $\lambda = 0.3$.

The hump shape illustrated in Figure 1 implies that the seller of good i has an incentive to make goods that are relatively simple too complex and goods that are relatively complex too simple. However, whether a good is relatively simple or complex in equilibrium (i.e., on the upward-sloping or downward-sloping segment of the $t_i(c_i)$ curve) is an equilibrium outcome that depends on all the parameters of the model.

Despite this, there is a simple way to determine whether a good is relatively simple or complex. Since $t_i(c_i)$ is hump-shaped, a given level of buyer attention t_i can be achieved in two ways: by choosing a low level of complexity or a high level of complexity. While the consumer allocates the same amount of attention to these two goods, the simpler one is well understood (high effective attention $\frac{t_i}{c_i}$), whereas

[9]The result that attention choice follows a hump shape holds even outside of the specific functional form assumed in this section. Under the assumptions laid out in Section 2, it is generally true that $t_i(c_i)$ is increasing for small c_i, decreasing for higher c_i and equal to zero above some level. The crucial assumption that leads to the buyer completely giving up at high levels of c_i is that v_i is bounded from below, which ensures that the marginal value of attention is bounded from above even when $t_i = 0$.

the more complex one is less well understood (low effective attention $\frac{t_i}{c_i}$). Indeed, goods with high effective attention lie on the upward-sloping part of the $t_i(c_i)$ curve, wheres goods that receive low effective attention are located on the downward-sloping part.

To see this formally, recall from Lemma 1 that $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$ has the same sign as $\frac{\partial t_i(c_i,\lambda)}{\partial c_i}\big|_\lambda$. From Equation (19), we know that the condition $\frac{\partial t_i(c_i,\lambda)}{\partial c_i}\big|_\lambda > 0$ can be rewritten as

$$\frac{(1-\theta_i)\cdot\delta_i\cdot w_i}{c_i\cdot 4\cdot\lambda} > 1 \tag{20}$$

Then, noting that we can rewrite Equation (19) as

$$\frac{t_i}{c_i} = 2\cdot\sqrt{\frac{(1-\theta_i)\cdot\delta_i\cdot w_i}{4\cdot c_i\cdot\lambda}} - 1, \tag{21}$$

we see that $\frac{\partial t_i(c_i,\lambda)}{\partial c_i}\big|_\lambda > 0$ if and only if $\frac{t_i}{c_i} > 1$. Therefore, sellers have an incentive to overcomplicate those goods that are relatively well understood by the consumer.

Proposition 4. Complexity and the Depth of Understanding. *When goods are ex ante heterogeneous, the seller has an incentive to*

- *overcomplicate goods that are relatively well understood in the planner's solution,* $\frac{t_i^*}{c_i^*} > 1$*;*
- *oversimplify goods that are not well understood in the planner's solution,* $\frac{t_i^*}{c_i^*} < 1$

Proposition 4 provides a simple characterization of the distortion of product complexity. Goods that in the planner's solution are relatively simple end up being too complex in equilibrium, whereas goods that in the planner's solution are complex end up being too simple (or dumbed down). This result stems from the fact that it is goods of intermediate complexity that attract the most attention from consumers: Simple goods are well understood even when little attention is paid to them, whereas very complex goods cannot be understood even if the consumer devotes all of her attention to them. To attract the consumer's attention, the seller therefore distorts complexity towards intermediate levels—

increasing the complexity of goods that should be relatively simple and decreasing the complexity fo goods that should be relatively complex.

Based on the above characterization, goods that are likely to be too complex include smartphones, checking accounts, and equity mutual funds. Most people invest significant time in using and understanding their smartphones. Producers seize on this through the development of new features and apps. In the case of checking accounts, our model predicts that banks have an incentive to add an inefficiently high number of contingent fees and promotional interest rates, which makes deposit contracts more complex than they should be. Similarly, actively managed mutual funds are generally much more complex than simple index funds, which most researchers would consider the optimal choice for investors. In contrast, our model implies that intricate policy debates may end up being oversimplified. For example, despite the apparent complications, the question of whether the UK should leave the EU was often dumbed down to how much the UK contributes to the EU budget.

5 Equilibrium and Comparative Statics

Up to now we have used first order conditions to characterize the seller's incentives to deviate from the socially optimal level of complexity. In this section, we characterize the equilibrium in full, starting with ex-ante homogeneous goods, and then extending the analysis to ex-ante heterogeneous goods.[10] In order to be able to solve for the equilibrium, we continue to use the functional form (17) for the value of the good.

5.1 Ex-Ante Homogeneous Goods

We first consider ex-ante homogeneous goods. In this case, the equilibrium first order condition (8) and the planner's first-order condition (11) can be written as stated in the following lemma.

[10]By ex-ante homogeneous we mean that the parameters that determine the functional form for v (i.e., w, δ, α, and θ) are equal across goods.

Lemma 2. First Order Conditions with Ex-Ante Homogeneous Goods. *Assume the value of good i takes the functional form (17). In a symmetric equilibrium with ex-ante homogenous goods, the equilibrium first-order condition is*

$$f'(c) = \left[\frac{T}{N \cdot c} - \frac{N-1}{2 \cdot N} \cdot \left(\frac{T}{N \cdot c} - 1\right)\right] \cdot \frac{\delta}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}, \tag{22}$$

whereas the planner's first order condition is

$$f'(c) = \frac{T}{N \cdot c} \cdot \frac{\delta}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}. \tag{23}$$

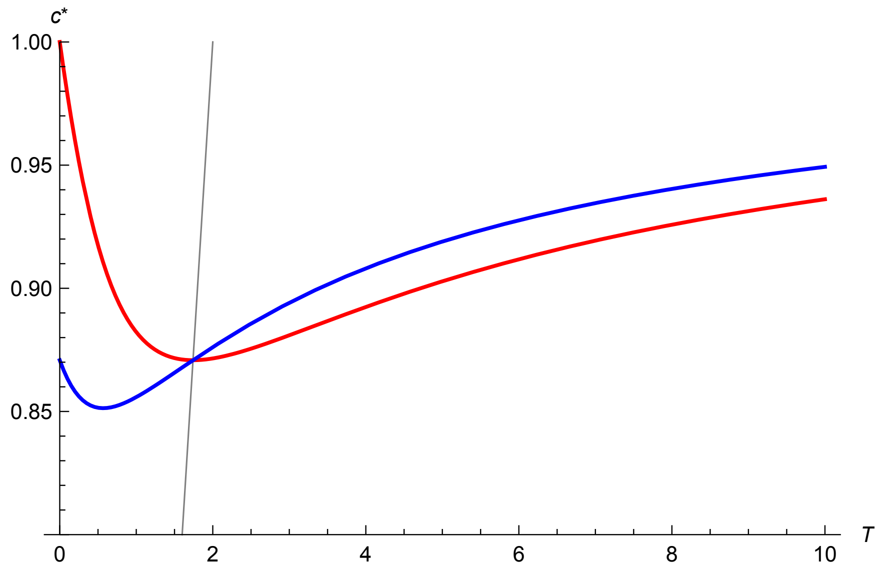
Under the quadratic benefit function (18), the marginal benefit of complexity is $f'(c) = \alpha - 2c$, so that solving for the equilibrium level of complexity requires solving a third-order polynomial. The solution can be expressed in closed form, but it is relatively complicated. We therefore present the key features of the equilibrium by comparing the relevant first-order conditions. In particular, comparing (22) and (23) shows that equilibrium complexity c^e is higher than planner's choice of complexity if and only if

$$c^e < \frac{T}{N}. \tag{24}$$

Condition (24) implicitly defines a separating hyperplane in the parameter space. On one side of this hyperplane ($c^e < \frac{T}{N}$) complexity is inefficiently high; on the other side ($c^e > \frac{T}{N}$) complexity is inefficiently low. The grey line in Figure 2 illustrates the separating hyperplane implied by (24) for the case of two goods. To the left of the grey line, equilibrium complexity c^e (the blue line) lies below the optimal level of complexity c^* (red line). To the right of the grey line, on the other hand, equilibrium complexity is higher than the optimal level of complexity.

One particularly interesting observation in Figure 2 is that, as the consumer's attention budget T increases, the equilibrium level of complexity (blue line) lies above that chosen by the planner (red line). Therefore, complexity rises to inefficiently high levels precisely when information processing

Figure 2: **Equilibrium and planner's optimal complexity as a function of attention capacity**



Homogenous goods, parameters: $N = 2$, $\delta = 0.9$, $w = 1$, $\alpha = 2$, $\theta = 0.5$. The blue line is the equilibrium choice, the red line the planner's choice. Both converge to the unconstrained optimal complexity of 1 as $T \to \infty$. The thin grey line is the separating hyperplane (here a line) between too low and too high complexity.

capacity grows. The figure therefore illustrates the *complexity paradox*: Rather than helping the buyer deal with complexity, increased information processing capacity can be a source of excessive complexity in the economy.[11] The following proposition formalizes this insight and establishes a number of additional comparative statics.

Proposition 5. The Complexity Paradox and Other Comparative Statics. *When goods are ex-ante homogenous and the benefit of understanding a good δ is not too large, all goods receive the same amount of attention. Equilibrium complexity is inefficiently high compared to the panner's solution if*

- *δ is large (i.e., it is the important to pay attention to the good);*
- *T is large (i.e., attention is abundant);*
- *α is small (i.e., the direct benefit of complexity is small);*

[11]This result confirms the intuition gained from equation (24): The first order effect of raising T is that it is more likely that equilibrium complexity c^e lies below the separating hyperplane. The adjustment of c^e in response to raising T does not overturn this first order effect.

- *N is small (i.e., there are fewer goods).*

The ranking of equilibrium and optimal complexity does not depend the seller's share of value, θ.

In addition to the complexity paradox discussed above, another interesting prediction of Proposition 5 is that increasing the number of goods that the buyer consumes (which one may argue is a natural byproduct of economic development) leads to goods that are overly simple. The reason is that, in symmetric equilibrium, all goods receive the same amount of attention, $\frac{T}{N}$. Therefore, more goods necessarily lead to less attention paid to each good (similar to decreasing the overall attention capacity T for a fixed number of goods). Therefore, using the same logic by which lower T leads to overly simplistic goods, so does increasing the number of goods N.

Note, however, that, while an increase in N leads to overly simple goods, it is not necessarily the case that goods become simpler in an absolute sense. In fact, Figure 2 illustrates that decreasing the attention budget T (equivalent to increasing N) can lead to an increase in complexity. The reason is that when attention is severely limited, goods are not understood very well no matter what, so that it becomes optimal to increase their complexity to gain some value in that dimension. Using a similar argument, a lower benefit from complexity α and a higher cost of inattention δ lead to too much complexity by lowering the equilibrium complexity c_i^e, making condition (24) more likely to hold.

Finally, note that Proposition 5 assumes that the benefit of understanding the good δ is not too large. This assumption ensures that the equilibrium is symmetric. When δ is large, there is potentially also an asymmetric equilibrium: For high δ and small T it can optimal (both in equilibrium and in the planner's problem) to choose complexity asymmetrically across otherwise identical goods: One good is very simple ($c_i = 0$) and receives no attention, whereas the other good is complex ($c_j > 0$) and receives all of the consumer's attention. Therefore, fundamentally similar products can have very different levels of complexity. As usual, the equilibrium does not pin down which good ends up being the complex one.

5.2 Ex-ante Heterogenous Goods

We now consider the case in which goods are allowed to differ in characteristics other than complexity (i.e., w, δ, α, and θ differ across goods). Equation (20) provides intuition for which goods tend to be well understood and therefore too complex: those that have a large utility weight (high w_i), those for which the buyer gets to keep more of the value (low θ_i), and those goods for which not paying attention is very costly (high δ_i). According to Equation (20), for these types of goods, the equilibrium level of complexity is likely to be on the upward-sloping part of the $\tilde{t}_i(c_i, \lambda)$ function. However, simply reading these results from equation (20) is not quite correct, because the equilibrium complexity c_i^e changes when the above parameters change. Proposition 6 therefore formalizes the above intuition in an equilibrium setting.

Table 1: **The Complexity Matrix**

	too simple	too complex
simple	low w_i $\sim$ less important good high θ_i $\sim$ seller's share high	high δ_i $\sim$ attention important low α_i $\sim$ complexity not beneficial
complex	low δ_i $\sim$ attention not important high α_i $\sim$ complexity beneficial	high w_i $\sim$ more important good low θ_i $\sim$ buyer's share high

Proposition 6. Heterogenous goods. *Assume that there are two goods ($N = 2$) that are initially identical in all parameters $\{w, \delta, \alpha, \theta\}$, and that at these parameters equilibrium complexity and planner's choice of complexity coincide. Introducing heterogeneity in one of the good-specific parameters $\pi \in \{w, \delta, \alpha, \theta\}$, such that for the first good $\pi_1 = \pi - \epsilon$ and for the second good $\pi_2 = \pi + \epsilon$, for small enough $\epsilon > 0$, the following holds:*

(i) *Utility weight ($\pi = w$). The good with the smaller utility weight w_i is simpler in an absolute sense and too simple relative to the planner's solution. The more important good (larger w_i) is more complex and too complex relative to the planner's solution.*

(ii) *Seller's share of value ($\pi = \theta$). The good for which the seller can extract a larger share (higher θ_i) is simpler in equilibrium and too simple relative to the planner's solution. The good for which sellers can extract less (lower θ_i) is more complex and too complex relative to the planner's solution.*

(iii) *Importance of attention ($\pi = \delta$). The good for which attention is less important (low δ_i) is more complex but too simple relative to the planner's solution. The good for which attention is more important (higher δ_i) is simpler but too complex relative to the planner's solution.*

(iv) *Direct benefit of complexity ($\pi = \alpha$). The good for which complexity has a larger direct benefit (higher α_i) is more complex but too simple relative to the planner's solution. The good for which complexity is less valuable (lower α_i) is simpler but too complex relative to the planner's solution.*

The results presented in Proposition 6 lead to a categorization of goods according to a complexity matrix illustrated in Table 1. The complexity matrix categorizes goods based on (i) whether they are simple or complex in an absolute sense and (ii) whether they are too simple or too complex relative to the planner's solution. As illustrated, depending on their characteristics goods can be categorized into four categories, according to their absolute and relative complexity. Therefore, while Proposition 4 provides a diagnostic as to whether goods are too complex or too simple based on the observed depth of understanding, Proposition 6 links both absolute and relative complexity back to the deep parameters of the model.

Above we argued that, based on the observation that smartphones are typically well understood by their users, they are likely too complex. But what drives this result? Using Table 1, we can identify a number of potential reasons. One the one hand, it could be the case that smartphones are too complex because attention is an important component of the value they generate (high δ). However, according to Table 1, in this case it must also be the case that smartphones are simple in an absolute sense. On the other hand, it could be the case that smartphones are too complex because either their utility weight w_i in the buyer's utility is high or because the buyer gets to keep most of the value (high $1-\theta_i$). In this case, Table 1 indicates that smartphones should be too complex as well as complex in an absolute sense. Absolute complexity is the amount of time needed to understand a good up to a certain level. With the assumed functional form for v_i, complexity c_i is equal to the amount of time t_i that has to be invested by the buyer to gain exactly half of the utility increase that one could get if one moved from not understanding the good at all ($t_i = 0$) to understanding the good perfectly ($t_i = \infty$). If one is willing to argue that it is relatively time consuming to learn to use a smartphone reasonably well, then we would be in the latter case: smartphones are complex in an absolute sense and also too complex relative to what a planner would choose.

6 Conclusion

Complexity is an important choice variable for the sellers of goods and financial products. Nonetheless, this complexity does not feature in most economic models, and when it does its only role is usually to obfuscate. This paper develops an equilibrium model of product complexity that explicitly allows for both the positive and negative side of complexity—complexity can generate value but it taxes the consumer's limited attention. The main result of our analysis is that, in a world in which buyers have limited attention, equilibrium complexity choices generally involve an *attention externality*: When choosing the complexity of their goods, sellers do not take into account that attention is a common resource that is shared across products. In equilibrium, sellers therefore distort the complexity of their products in order to divert attention from the goods of other sellers. Depending on the consumer's reaction to an increase in complexity—does the consumer devote more or less time when a product becomes more complex?— this can lead to *too much or too little complexity* in equilibrium.

Our model yields a number of interesting insight. Perhaps surprisingly, the goods that are likely to be excessively complex are those that, in equilibrium, are relatively well understood by the consumer.

The reason is that it is the consumer's willingness to spend time to understand the good that induces the seller to raise the good's complexity above the efficient level. We also show that, paradoxically, equilibrium complexity is more likely to be excessive when attention is abundant. Therefore, rather than helping buyers deal with complexity, increases in information processing capacity make it more likely that complexity is excessive—the *complexity paradox*. Finally, with heterogeneous goods, our model allows us to characterize the factors that determine which goods are simple and which complex, both in an absolute sense and relative to the efficient level of complexity.

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A Proofs

Proof of Proposition 1.

What the incentives of seller i to change the level of complexity c_i of good i from the planner's optimum $(c_1^*, ...c_N^*)$ depends on the difference between the seller's first-order condition (8) and that of the planner (11), both evaluated at the planner's choice $(c_1^*, ...c_N^*)$. We rewrite the difference between the right-hand side of the two first-order conditions as

$$\sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{\partial t_j(c_1^*, ..., c_N^*)}{\partial c_i} = \sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{d\tilde{t}_j(c_j^*, \lambda(c_1^*, ..., c_N^*))}{dc_i} = \frac{\partial \lambda(c_1^*, ..., c_N^*)}{\partial c_i} \cdot \sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{\partial \tilde{t}_j(c_j^*, \lambda^*)}{\partial \lambda}, \quad \text{(A1)}$$

where we use t_i^* as a shorthand for $t_i(c_1^*, ..c_N^*)$ and λ^* denotes the shadow cost of attention at the planner's solution, $\lambda(c_1^*, ..c_N^*)$. The first step in equation (A1) uses the fact that we can rewrite the buyers attention choice $t_j(c_1^*, ..c_N^*)$ as a function of the complexity of good j and the shadow cost of attention λ as $\tilde{t}_j(c_j^*, \lambda(c_1^*, ..c_N^*))$. The second step applies the chain rule, $\frac{d\tilde{t}_j(c_j^*, \lambda(c_1^*, ..c_N^*))}{dc_i} = \frac{\partial \lambda(c_1^*, ..c_N^*)}{\partial c_i} \frac{\partial \tilde{t}_j(c_j^*, \lambda^*)}{\partial \lambda}$ and moves the first (constant) term outside of the summation sign.

Note that raising the shadow cost of attention leads to less attention being paid to all goods, because the inverse function theorem implies

$$\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} = \frac{1}{\frac{\partial \tilde{v}_i^2(c_i, \tilde{t}_i(c_i, \lambda))}{\partial \tilde{t}_i^2}} < 0, \quad \text{(A2)}$$

where we used $\tilde{v}_j(c_j, \tilde{t}_j) = v_j(c_j, \frac{t_j}{c_j})$ and $\frac{\partial \tilde{v}_j^2(c_j, \tilde{t}_j(c_j, \lambda))}{\partial \tilde{t}_j^2} < 0$ by assumption.

Thus from (A1), the externality is negative if $\frac{\partial t_i(c_1^*, ..c_N^*)}{\partial c_i} > 0$, meaning that the panner's optimum must entail a lower level of c_i, which in turn increases the left hand side of (11) (due to the decreasing benefits of complexity we have assumed). □

Proof of Lemma 1.

Attention t_i allocated to good i can be implicitly expressed from (4) using $\tilde{t}_i(c_i, \lambda)$. Attention grabbing $\frac{\partial t_i(c_1, ..c_N)}{\partial c_i}$ can be written as:

$$\frac{\partial t_i(c_1, ..c_N)}{\partial c_i} = \frac{d\tilde{t}_i(c_i, \lambda(c_1, ..c_N))}{dc_i} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda(c_1, ..c_N)}{\partial c_i} \quad \text{(A3)}$$

where the first term is the effect of c_i on t_i keeping λ fixed, while the second the indirect effect through λ.

The defining implicit equation for equilibrium $\lambda(c_1, ..c_N)$ is:

$$T = \sum_j t_j = \sum_j \tilde{t}_j (c_j, \lambda) \tag{A4}$$

Without a specific functional form for v we cannot express $\lambda(c_1, ..c_N)$ explicitly in the heterogenous setup but we can take the derivative of interest $\frac{d.}{dc_i}$ of (A4):

$$0 = \frac{\partial \tilde{t}_i (c_i, \lambda)}{\partial c_i} + \sum_{j=1}^{N} \frac{\partial \tilde{t}_j (c_j, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial c_i}, \tag{A5}$$

from which one can express

$$\frac{\partial \lambda}{\partial c_i} = \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}. \tag{A6}$$

Plugging this into (A3) yields

$$\frac{\partial t_i(c_1, ..c_N)}{\partial c_i} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} \cdot \left[1 - \frac{-\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}\right] \tag{A7}$$

The second term is positive as $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} < 0$ for all j (see (A2)). Thus $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}$ has the same sign as $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$. By the same argument, it is obvious from (A6) that $\frac{\partial \lambda}{\partial c_i}$ also has the same sign as $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$.

Now we turn to $\frac{\partial^2 \tilde{v}(c_i,t_i)}{\partial c_i \partial t_i}$. One can rewrite the buyers problem (2) in terms of $\tilde{v}$:

$$\max_{t_1,..t_N} \sum_{i=1}^{N} (1 - \theta_i) \cdot \tilde{v}_i (c_i, t_i). \tag{A8}$$

which is maximized subject to (3) and we get the counterpart of (4) wich for interior solution of t_i can be written as:

$$\frac{1}{(1 - \theta_i)} \cdot \lambda = \frac{\partial \tilde{v}_i (c_i, t_i)}{\partial t_i} \tag{A9}$$

Here the seller treats c_i as given. What happens if we allow this predetermined c_i to vary? Taking the partial with respect to c_i while keeping all other c's as fixed:

$$\frac{1}{1 - \theta_i} \cdot \frac{\partial \lambda}{\partial c_i} = \frac{\partial^2 \tilde{v}_i (c_i, t_i)}{\partial c_i \partial t_i} + \frac{\partial^2 \tilde{v}_i (c_i, t_i)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial^2 \tilde{v}_i (c_i, t_i)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial c_i} \tag{A10}$$

where we took into account that t_i is a function of c_i and λ, thus $\tilde{t}_i(c_i,\lambda)$ and at the same time λ is the function of all other c's. Using (A2) that $\frac{\partial^2 \tilde{v}_i(c_i,t_i)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} = 1$ and rearranging:

$$\frac{\theta_i}{1-\theta_i} \cdot \frac{\partial \lambda}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{1}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}. \tag{A11}$$

Using (A6) to substitute $\frac{\partial \lambda}{\partial c_i}$ one arrives at

$$\frac{\theta_i}{1-\theta_i} \cdot \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{1}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}. \tag{A12}$$

Rearranging again yields:

$$\frac{\frac{\theta_i}{1-\theta_i} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} + \sum_{j=1}^{N} \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} \cdot \sum_{j=1}^{N} \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i}, \tag{A13}$$

since $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} < 0$ for all $i \in \{1,..N\}$, it follows that $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}$ and $\frac{\partial^2 \tilde{v}_i(c_i,t_i)}{\partial c_i \partial t_i}$ have the same sign. □

Proof of Proposition 2.

See text. □

Proof of Proposition 3.

As shown in Proposition 1, the direction of the complexity distortion is determined by the sign of $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$. Lemma 1 shows that this distortion has the same sign as $\frac{\partial \lambda}{\partial c_i}$. Therefore, sellers have an incentive to distort complexity in the direction that raises the shadow price of attention λ. Thus starting from the social planner's solution and allowing sellers to update (e.g. following a best response dynamics), in every step of the iteration, the shadow cost of attention weakly increases (strictly if $\frac{\partial t_i(c_1,..c_N)}{\partial c_i} > 0$). Thus $\lambda^e > \lambda^*$ whenever equilibrium complexity and the planner's solution do not coincide (which is the only case in which $\frac{\partial t_i(c_1,..c_N)}{\partial c_i} = 0$ for all sellers). □

Proof of Proposition 4.

See text. □

Proof of Lemma 2.

We use the heterogenous setup until noted otherwise. The first-order condition of the buyer for a given λ is:

$$\tilde{t}_i(c_i, \lambda) = \sqrt{\frac{c_i \cdot \delta_i \cdot (1-\theta_i) \cdot w_i}{\lambda}} - c_i. \tag{A14}$$

Plugging the above into (3) that time adds up to T one can express λ:

$$\lambda = \frac{\left(\sum_{k=1}^{N} \sqrt{c_k \delta_k (1-\theta_k) w_k}\right)^2}{\left(\sum_{j=1}^{N} c_j + T\right)^2}, \tag{A15}$$

substituting this back into (A14) we arrive at:

$$t_i(c_1, ..c_N) = \frac{\sqrt{c_i \delta_i (1-\theta_i) w_i}}{\sum_{k=1}^{N} \sqrt{c_k \delta_k (1-\theta_k) w_k}} \cdot \left(\sum_{j=1}^{N} c_j + T\right) - c_i. \tag{A16}$$

Taking the derivative of interest that captures the attention grabbing:

$$\frac{\partial t_i(c_1, ..c_N)}{\partial c_i} = \frac{\sum_{j \neq i} \sqrt{c_j \delta_j (1-\theta_j) w_j}}{\left(\sum_{k=1}^{N} \sqrt{c_k \delta_k (1-\theta_k) w_k}\right)^2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{c_i}} \cdot \sqrt{\delta_i (1-\theta_i) w_i} \cdot \left(\sum_{j=1}^{N} c_j + T\right) + \frac{\sqrt{c_i \delta_i (1-\theta_i) w_i}}{\sum_{k=1}^{N} \sqrt{c_k \delta_k (1-\theta_k) w_k}} - 1. \tag{A17}$$

Imposing symmetry one gets:

$$\frac{\partial t}{\partial c} = \frac{N-1}{2 \cdot N} \cdot \left(\frac{T}{N \cdot c} - 1\right) \tag{A18}$$

and

$$\lambda = \frac{w \cdot \delta \cdot (1-\theta)}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}. \tag{A19}$$

Plugging these into (8) and (12) and using that the for the symmetric case with the given functional form $\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = w \cdot f'(c)$ and that in a symmetric equilibrium all goods get the same amount of attention $t = \frac{T}{N}$ we arrive at the equations stated in the lemma. □

Proof of Proposition 5.

The statement is made for low enough $\delta < \delta^*$, because if δ is too high then the planner wants to differentiate goods. In the following we calculate a lower bound for δ^*. First note that the maximum v that can be derived from a good is when the complexity is chosen to be its first best (unconstrained) optimum of $c_i = \frac{\alpha_i}{2}$ and effective attention is chosen to be infinite $\frac{t_i}{c_i} = \infty$. Thus:

$$v_i\left(c_i, \frac{t_i}{c_i}\right) < v_i\left(\frac{\alpha_i}{2}, \infty\right) = w_i \cdot \left(\delta_i + \frac{\alpha_i^2}{4}\right) \tag{A20}$$

Also with N ex ante symmetric goods if all goods have the same amount of complexity and thus get the same amount of attention from the buyer, then v_i can be bounded from below because if $c = \frac{\alpha}{2}$ is not optimal, then

v would be higher with the optimal choice:

$$v_i\left(c_i, \frac{t_i}{c_i}\right) > v\left(\frac{\alpha}{2}, \frac{T/N}{\alpha/2}\right) = \frac{w\left(\alpha^3 N + 2T\left(\alpha^2 + 4\delta\right)\right)}{4\alpha N + 8T} \tag{A21}$$

We need to show that

$$N \cdot v\left(c_s^*, \frac{t_{s,s}^*}{c_s^*}\right) > (N-1) \cdot v\left(c_a^*, \frac{t_{s,a}^*}{c_a^*}\right) + 1 \cdot v(0, \infty) \tag{A22}$$

where c_s^* is the panner's optimum in the symmetric case and c_a^* in the asymmetric one (in which one of the goods has zero complexity but the others have the same). A sufficient condition for the symmetric solution is that

$$\frac{w\left(\alpha^3 N + 2T\left(\alpha^2 + 4\delta\right)\right)}{4\alpha N + 8T} > (N-1) \cdot w \cdot \left(\delta + \frac{\alpha^2}{4}\right) + w \cdot \delta \tag{A23}$$

which simplifies to

$$\delta < \frac{\alpha(\alpha N + 2T)}{4N^2} \tag{A24}$$

and holds for small enough δ and is more likely to be violated for small attention capacity T. It holds for all T (including $T = 0$) if $\delta < \frac{\alpha^2}{4 \cdot N}$: this is the sufficient (but not necessary) condition for the panner's optimum to be symmetric.

The separating hyperplane for which the optimal and equilibrium levels of complexity are the same happens at critical attention level T at which all goods have complexity $c = \frac{T}{N}$ (see Eq. 24). Plugging this into Eq. 11 yields a quadratic equation for T, for which the only solution that corresponds to a maximum of the social welfare function is (this can be checked by signing the second order condition):

$$T^{crit} = \frac{N}{4}\left(\alpha + \sqrt{\alpha^2 - 2\delta}\right) \tag{A25}$$

This defines a separating hyperplane in the parameter space, so by continuity of equilibrium complexity in the parameters, all we have to check is whether there is too much complexity on one side of the hyperplane arbitrarily close to the hyperplane itself.

Taking the total derivative of the two first order conditions Eq. 8 and 11 with respect to δ and substituting $c = \frac{T}{N}$ (at $c^e = c^*$), and expressing $c^{e\prime}(\delta)$ and $c^{*\prime}(\delta)$ the two FOCs yield:

$$c^{e\prime}(\delta) = \frac{2NT}{\delta N(N+1) - 16T^2} \tag{A26}$$

$$c^{*\prime}(\delta) = \frac{NT}{\delta N^2 - 8T^2} \tag{A27}$$

We want to show that if δ is a bit higher than on the hyperplane, then equilibrium complexity is higher than the panner's optimum. Thus we have to show $c^{e\prime}(\delta) > c^{*\prime}(\delta)$ at $T = T^{crit}$ which holds if,

$$\frac{\sqrt{\alpha^2 - 2\delta} + \alpha}{\left(\alpha\sqrt{\alpha^2 - 2\delta} + \alpha^2 - 2\delta\right)\left(N\left(2\alpha\sqrt{\alpha^2 - 2\delta} + 2\alpha^2 - 3\delta\right) - \delta\right)} \geq 0, \tag{A28}$$

which holds if $\delta < \frac{\alpha^2}{2}$.

Taking the total derivative of the two first order conditions Eq. 8 and 11 with respect to T and substituting $c = \frac{T}{N}$ (at $c^e = c^*$), and expressing $c^{e\prime}(T)$ and $c^{*\prime}(T)$ the two FOCs yield:

$$c^{e\prime}(T) = \frac{\delta - \delta N}{\delta N(N+1) - 16T^2} \tag{A29}$$

$$c^{*\prime}(T) = 0 \tag{A30}$$

$c^{e\prime}(T) > c^{*\prime}(T)$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$.

Taking the total derivative of the two first order conditions Eq. 8 and 11 with respect to α and substituting $c = \frac{T}{N}$ (at $c^e = c^*$), and expressing $c^{e\prime}(\alpha)$ and $c^{*\prime}(\alpha)$ the two FOCs yield:

$$c^{e\prime}(\alpha) = \frac{8T^2}{16T^2 - \delta N(N+1)} \tag{A31}$$

$$c^{*\prime}(\alpha) = \frac{4T^2}{8T^2 - \delta N^2} \tag{A32}$$

$c^{e\prime}(\alpha) < c^{*\prime}(\alpha)$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$.

Taking the total derivative of the two first order conditions Eq. 8 and 11 with respect to N and substituting $c = \frac{T}{N}$ (at $c^e = c^*$), and expressing $c^{e\prime}(N)$ and $c^{*\prime}(N)$ the two FOCs yield:

$$c^{e\prime}(N) = \frac{\delta(N-1)T}{N\left(\delta N(N+1) - 16T^2\right)} \tag{A33}$$

$$c^{*\prime}(N) = 0 \tag{A34}$$

$c^{e\prime}(N) < c^{*\prime}(N)$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$. □

Proof of Proposition 6.

Heterogeneity in δ: $\delta_1 = \delta - \epsilon$ for good 1 and $\delta_2 = \delta + \epsilon$ for good 2, where δ is the level at which the equilibrium and planner's levels of complexity coincide for both goods. We know from (24) that the level of complexity for both goods at $\epsilon = 0$ is very simple

$$c_1^s = c_2^s = c_1^p = c_2^p = \frac{T}{2}. \tag{A35}$$

The strategy of the proof is to look at the derivative of equilibrium and planner's optimal complexity in ϵ. This way we can conclude which complexity is higher if ϵ is small enough. We can express these derivatives by taking total differential of the first-order conditions for the two goods and then setting $\epsilon = 0$: Equation 7 in the equilibrium case and Equation 10 in the planner's case. In the equilibrium, this gives us two equations and we can solve for the two unknowns: $\left.\frac{\partial c_1^p}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_1^p}{\partial \delta_1}\right|_{\delta_1=\delta}$ and $\left.\frac{\partial c_2^p}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_2^p}{\partial \delta_2}\right|_{\delta_2=\delta}$. Similarly we have two equations and two unknowns for the derivatives of the planner's optimal complexity $\left.\frac{\partial c_1^s}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_1^s}{\partial \delta_1}\right|_{\delta_1=\delta}$ and $\left.\frac{\partial c_2^s}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_2^s}{\partial \delta_2}\right|_{\delta_2=\delta}$.

First note that in all cases it must be the case that the parameters are chosen s.t. the level of T is equal to (A25) because this is the combination of parameters at equilibrium and planner's levels of complexity coincide.

The derivatives of interest in the panner's optimum are

$$\left.\frac{\partial c_1^s}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^s}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{T}{2\left(2T^2 - \delta\right)}, \tag{A36}$$

for the equilibrium

$$\left.\frac{\partial c_1^p}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{3T}{2\left(8T^2 - 3\delta\right)}. \tag{A37}$$

If $T > \sqrt{\frac{3}{8}\delta}$ then it follows that

$$\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_1^p}{\partial \theta_1}\right|_{\theta_1=\theta} \tag{A38}$$

thus if ϵ is small enough then good 2 (with the higher δ) is simpler than good 1 (with the lower δ). $T > \sqrt{\frac{3}{8}\delta}$ holds since T is defined by (A25) (with $N = 2$) and $\alpha^2 - 2\delta > 0$ follows from the existence (assumed in the statement of the theorem) of a critical T where the planner's and equilibrium complexity coincide.

If furthermore $T > \frac{\sqrt{\delta}}{\sqrt{2}}$ (which again follows from (A25)), it is straightforward to show that

$$\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} > \left.\frac{\partial c_2^s}{\partial \theta_2}\right|_{\theta_2=\theta}. \tag{A39}$$

This proves that if ϵ is small enough, then good 2 (with the higher δ) is too complex in equilibrium than in the planner's choice. By the same logic this proves that good 1 (with the lower δ) is too simple.

The rest of the proofs follow a similar logic and we just report the main steps. We also assume in all the following that $T > \frac{\sqrt{\delta}}{\sqrt{2}}$.

Heterogeneity in θ: The derivatives of interest in the panner's optimum are

$$\left.\frac{\partial c_1^s}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^s}{\partial \theta_2}\right|_{\theta_2=\theta} = 0, \tag{A40}$$

for the equilibrium

$$\left.\frac{\partial c_1^p}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{\delta T}{2(1-\theta)\left(8T^2 - 3\delta\right)}. \tag{A41}$$

It follows that

$$\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_1^p}{\partial \theta_1}\right|_{\theta_1=\theta} \tag{A42}$$

and

$$\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_2^s}{\partial \theta_2}\right|_{\theta_2=\theta}. \tag{A43}$$

Heterogeneity in α: The derivatives of interest in the panner's optimum are

$$\left.\frac{\partial c_1^s}{\partial \alpha_1}\right|_{\alpha_1=\alpha} = -\left.\frac{\partial c_2^s}{\partial \alpha_2}\right|_{\alpha_2=\alpha} = -\frac{T^2}{2T^2 - \delta}, \tag{A44}$$

for the equilibrium

$$\left.\frac{\partial c_1^p}{\partial \alpha_1}\right|_{\alpha_1=\alpha} = -\left.\frac{\partial c_2^p}{\partial \alpha_2}\right|_{\alpha_2=\alpha} = -\frac{4T^2}{8T^2-3\delta}. \tag{A45}$$

It follows that

$$\left.\frac{\partial c_2^p}{\partial \alpha_2}\right|_{\alpha_2=\alpha} > \left.\frac{\partial c_1^p}{\partial \alpha_1}\right|_{\alpha_1=\alpha} \tag{A46}$$

and

$$\left.\frac{\partial c_2^p}{\partial \alpha_2}\right|_{\alpha_2=\alpha} < \left.\frac{\partial c_2^s}{\partial \alpha_2}\right|_{\alpha_2=\alpha}. \tag{A47}$$

Heterogeneity in w: The derivatives of interest in the panner's optimum are

$$\left.\frac{\partial c_1^s}{\partial w_1}\right|_{w_1=w} = -\left.\frac{\partial c_2^s}{\partial w_2}\right|_{w_2=w} = \frac{T\left(\delta+2T^2-2\alpha T\right)}{2w\left(2T^2-\delta\right)}, \tag{A48}$$

for the equilibrium

$$\left.\frac{\partial c_1^p}{\partial w_1}\right|_{w_1=w} = -\left.\frac{\partial c_2^p}{\partial w_2}\right|_{w_2=w} = \frac{T\left(3\delta+8T^2-8\alpha T\right)}{2w\left(8T^2-3\delta\right)}. \tag{A49}$$

It follows that

$$\left.\frac{\partial c_2^p}{\partial w_2}\right|_{w_2=w} > \left.\frac{\partial c_1^p}{\partial w_1}\right|_{w_1=w} \tag{A50}$$

and

$$\left.\frac{\partial c_2^p}{\partial w_2}\right|_{w_2=w} > \left.\frac{\partial c_2^s}{\partial w_2}\right|_{w_2=w}. \tag{A51}$$

For the first we need that $T \in \left(\sqrt{\frac{3}{8}}\sqrt{\delta}, \frac{1}{2}\left(\sqrt{\alpha^2-\frac{3\delta}{2}}+\alpha\right)\right)$, while for the second we need $T > \frac{\alpha}{2}$, both follow from (A25). □